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## GUJARAT TECHNOLOGICAL UNIVERSITY <br> BE - SEMESTER - VI (NEW).EXAMINATION - WINTER 2016

Subject Code: 2160704
Subject Name: Theory of Computation Time: 10:30 AM to 01:00 PM Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) Use the principle of mathematical induction to prove that
$1+3+5+\ldots+r=n^{2} \quad$ for all $n>0$ where $r$ is an odd integer $\& n$ is the number of terms in the sum. ( Note : $\mathrm{r}=2 \mathrm{n}-1$ )
(b) Convert the CFG, $\mathrm{G}(\{\mathrm{S}, \mathrm{A}, \mathrm{B}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{P}, \mathrm{S})$ to CNF, where P is as follows S --> $\mathrm{aAbB} \quad \mathrm{A}$--> $\mathrm{Ab} \mid \mathrm{b} \quad \mathrm{B}$--> $\mathrm{Ba} \mid \mathrm{a}$
Q. 2 (a) Draw a Turing Machine(TM) to accept Palindromes over $\{\mathrm{a}, \mathrm{b}\}$. (Even as well as Odd Palindromes)
(b) Convert the NFA given in Table below to its corresponding DFA and draw the DFA .

| Current State | Input symbol |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $\rightarrow \mathrm{Q}_{0}$ | $\mathrm{Q}_{1}$ | $\mathrm{Q}_{0}, \mathrm{Q}_{2}$ |
| $\mathrm{Q}_{1}$ | $\mathrm{Q}_{2}$ | $\mathrm{Q}_{0}$ |
| $\mathrm{Q}_{2}{ }^{*}$ | Q 0 | --- |

(b) Prove that the following CFG is Ambiguous.

$$
\mathbf{S}->\mathbf{S}+\mathbf{S}|\mathbf{S} * \mathbf{S}| \mathbf{a} \mid \mathbf{b}
$$

Write the unambiguous CFG based on precedence rules for the above grammar. Derive the parse tree for expression $(a+a) * b$ from the unambiguous grammar.
Q. 3 (a) Let $A=\{1,2,3,4,5,6\}$ and $R$ be a relation on $A$ such that $a R b$ iff $a$ is a multiple of $b$. Write R. Check if the relation is i) Reflexive ii) Symmetric iii) Asymmetric iv) Transitive
(b) There are 2 languages over $\sum=\{a, b\}$
$\mathrm{L} 1=$ all strings with a double " a "
$\mathrm{L} 2=$ all strings with an even number of " $a$ "
Find a regular expression and an FA that define L1 $\cap \mathrm{L} 2$
OR
Q. 3 (a) If $L=\left\{0^{i} 1^{i} \mid i \geq 0\right\}$ Prove that $L$ is regular. 07
(b) Prove that if L 1 and L 2 are regular languages then $\mathrm{L} 1 \cap \mathrm{~L} 2$ is also a regular language.
Q. 4 (a) Given a CFG , $\mathrm{G}=(\{\mathrm{S}, \mathrm{A}, \mathrm{B}\},\{0,1\}, \mathrm{P}, \mathrm{S})$ with P as follows

S --> 0B| 1A A --> $0 S|1 \mathrm{AA}| 0 \quad \mathrm{~B}$--> $1 \mathrm{~S}|0 \mathrm{BB}| 1$
Design a PDA M corresponding to CFG, G. Show that the string 0001101110 belongs to CFL, L(G)
(b) Design a PDA, $M$ to accept $L=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{2 \mathrm{n}} \mid \mathrm{n} \geq 1\right\} \quad \mathbf{0 7}$
Q. 4 (a) Design a FA for the regular expression $(0+1)(01)^{*}(011)^{*}$
(b) Write a regular expression for language $L$ over $\{0,1\}$ such that every string in $L \quad \mathbf{0 7}$
i) Begins with 00 and ends with 11.
ii) Contains alternate 0 and 1 .
Q. 5 (a) Draw a transition diagram for a Turing machine accepting the following language. $\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}} \mathrm{c}^{\mathrm{n}} \mid \mathrm{n} \geq 0\right\}$
(b) Explain Universal Turing machine with the help of an example
Q. 5 (a) Define functions by Primitive Recursion. Show that the function $f(x, y)=x+y$ is primitive recursive.
(b) Prove Kleene's Theorem (Part I): Any Regular Language can be accepted by a Finite Automaton (FA).

