## GUJARAT TECHNOLOGICAL UNIVERSITY <br> BE - SEMESTER-VI • EXAMINATION - WINTER 2013

Subject Code: 160704
Date: 06-12-2013

## Subject Name: Theory of Computation

 Time: 02:30 pm to 05:00 pmTotal Marks: 70 Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 Answer the following.
(a) Write Regular Expressions for the following languages of all strings in \{0,1\}*
(i) Strings that start with 1 and do not end with 10 .
(ii) Strings with length 6 or less.
(b) Prove that $\sqrt{ } \mathbf{2}$ (square root of 2 ) is Irrational by method of Contradiction.
(c) Design a CFG(Context Free Grammar) for the following language.
$\mathrm{L}=\left\{\mathbf{0}^{\mathbf{i}} \mathbf{1}^{\mathrm{j}} \mathbf{0}^{\mathrm{k}} / \mathrm{j}>\mathrm{i}+\mathrm{k}\right\}$
(d) Explain one-to-one, onto and bijection function with suitable example.
Q. 2 (a) Using Principle of Mathematical Induction, Prove that for every $\mathrm{n}>=1$,

$$
7+13+19+\ldots+(6 n+1)=n(3 n+4)
$$

(b) Compare FA, NFA and NFA- $\Lambda$.

For the following Regular Expression draw an NFA- $\Lambda$ recognizing the corresponding language.
$(0+1)^{*}(10+110)^{*} 1$
OR
(b) Prove Kleene's Theorem (Part I): Any Regular Language can be accepted by a Finite Automaton(FA) .
Q. 3 (a) Convert following NFA- $\Lambda$ to NFA and FA.

| $\mathbf{Q}$ | $\boldsymbol{\delta}(\mathbf{q}, \mathbf{\Lambda})$ | $\boldsymbol{\delta}(\mathbf{q}, \mathbf{0})$ | $\boldsymbol{\delta}(\mathbf{q}, \mathbf{1})$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{A}$ | $\{\mathbf{B}, \mathbf{D}\}$ | $\{\mathbf{A}\}$ | $\dot{\boldsymbol{\emptyset}}$ |
| $\mathbf{B}$ | $\dot{\boldsymbol{\emptyset}}$ | $\{\mathbf{C}\}$ | $\{\mathbf{E}\}$ |
| $\mathbf{C}$ | $\dot{\boldsymbol{\emptyset}}$ | $\dot{\boldsymbol{\emptyset}}$ | $\{\mathbf{B}\}$ |
| $\mathbf{D}$ | $\dot{\boldsymbol{\emptyset}}$ | $\{\mathbf{E}\}$ | $\{\mathbf{D}\}$ |
| $\mathbf{E}$ | $\dot{\boldsymbol{\emptyset}}$ | $\dot{\boldsymbol{\emptyset}}$ | $\dot{\boldsymbol{\emptyset}}$ |

(b) Draw FA for accepting:
(i)The string in $\{0,1\}^{*}$ ending in 1 and not containing substring 00 .
(ii)The strings with odd no. of 1's and odd no. of 0's

## OR

Q. 3 (a) Draw Finite Automata (FA) for following languages:
$\mathrm{L}_{1}=\left\{\mathrm{x} / 11\right.$ is not a substring of $\left.\mathrm{x}, \mathrm{x} \in\{0,1\}^{*}\right\}$
$L_{2}=\left\{x / x\right.$ ends with $\left.10, x \in\{0,1\}^{*}\right\}$
Find FA accepting languages (i) $L_{2}-L_{1}$ and (ii) $L_{1} \cap L_{2}$
(b) Prove that the following CFG is Ambiguous.
$\mathbf{S} \rightarrow \mathbf{S}+\mathbf{S}|\mathbf{S} * \mathbf{S}| \mathbf{S}) \mid \mathbf{a}$
Write the unambiguous CFG for the above grammar. Draw Parse tree for the string $a+a * a$.
Q. 4 (a) For the language $\mathrm{L}=\left\{\mathrm{xcx}^{\mathrm{r}} / \mathrm{x} \in\{\mathrm{a}, \mathrm{b}\}^{*}\right\}$ (Palindrome with middle character $=\mathrm{c}$ ), Design a PDA(Push Down Automata) and trace it for string "abacaba".
(b) Convert following CFG to equivalent Chomsky Normal Form(CNF).
$\mathrm{S} \rightarrow \mathrm{AACD}|\mathrm{ACD}| \mathrm{AAC}|\mathrm{CD}| \mathrm{AC} \mid \mathrm{C}$
$\mathrm{A} \rightarrow \mathrm{aAb} \mid \mathrm{ab}$
$\mathrm{C} \rightarrow \mathrm{aCla}$
$\mathrm{D} \rightarrow \mathrm{aDa}|\mathrm{bDb}| \mathrm{aa} \mid \mathrm{bb}$

## OR

Q. 4 (a) Design and draw a deterministic PDA accepting strings of the language
$\mathrm{L}=\left\{\mathrm{x} \in\{\mathrm{a}, \mathrm{b}\}^{*} \mid \mathrm{n}_{\mathrm{a}}(\mathrm{x})>\mathrm{n}_{\mathrm{b}}(\mathrm{x})\right\}$. Trace it for the string "aababaab".
(b) Define Turing Machine. Draw a Turing Machine(TM) to accept

Palindromes over $\{\mathrm{a}, \mathrm{b}\}$. (Even as well as Odd Palindromes)
Q. 5 Write short notes on the following (Any FOUR)
(i) Equivalence Relation.
(ii) Universal Turing Machine.
(iii) The Sets P, NP, PSpace and NPSpace.
(iv) The Primitive Recursive Functions.
(v) Application of Pumping Lemma.
(vi) Basic Complexity Classes.

